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Small subhypermodules and their applications

Behnam Talaee
Department of Mathematics, Faculty of Basic Sciences,
Babol University of Technology, Babol, Iran
behnamtalaee@nit.ac.ir

Abstract

Let R be a hyperring (in the sense of [5]) and M be a hypermodule on R. In this article we introduce class of small subhypermodules of M. First we get some properties of subhypermodules and then the class of small subhypermodules and small homomorphism in the category of hypermodules are investigated. For example we show that if M is a hypermodule and N is a direct summand of M, then a small subhypermodule K of M which is contained in N, is small in N. Also we get some important applications of small subhypermodules in category of hypermodules (for example in exact sequences etc.).

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1 Introduction

The categories of hypergroups, hypermodules and hyperrings have many important roles in hyperstructures. Some authors got many exiting results about these theories. Reader can see references [1], [3], [4], [5] to get some basic information about the categories of *hypergroups*, *hyperrings* and *hypermodules*. Also reference [8] can be suitable to get some information about rings and modules theory.
We recall some definitions and theorems from above references which we need them to develop our paper.

A *hyperstructure* is a nonvoid set H together with a function $. : H \times H \longrightarrow P^*(H)$, where . is called a hyperoperation and $P^*(H)$ is the set of all nonempty subsets of H.

For $A, B \subseteq H$ and $x \in H$ we define

$$A.B = \bigcup_{a \in A, b \in B} a.b, \qquad x.B = \{x\}.B, \qquad A.x = A.\{x\}.$$

Definition 1.1 A hyperstructure H with a hyperoperation $+$ is called a *canonical hypergroup* if the following hold for H;

(i) $(x + y) + z = x + (y + z)$ for all $x, y, z \in H$;

(ii) $x + y = y + x$ for all $x, y \in H$;

(iii) there is an element, say 0, such that $0 + x = \{x\}$, for every $x \in H$;

(iv) For each $x \in H$ there exists a unique element $x' \in H$, such that $0 \in x + x'$. (we denote x' by $-x$ and it is called the opposite of x). Also we write $x - y$ instead of $x + (-y)$;

(v) $z \in x + y \Longrightarrow y \in z - x$ for all x, y, z in H.

Note that 0 is unique and for every $x \in H$ we have $x + 0 = 0 + x = \{x\}$, we identify a singleton set $\{x\}$ by x.
Canonical hypergroups were studied by J. Mittas in [7].

Definition 1.2 A non-void set R with a hyperoperation $(+)$ and with a binary operation $(.)$ is called a *hyperring* if

(R_1) : $(R, +)$ is a canonical hypergroup;

(R_2) : $(R, .)$ is a multiplicative semigroup having 0, such that $x.0 = 0.x = 0$ for all $x \in R$;

(R_3) : $z.(x + y) = z.x + z.y$ and $(x + y).z = x.z + y.z$ for all $x, y, z \in R$.

If there exists an element $1 \in R$ such that $1.x = x.1 = x$ for all $x \in R$, then we say R is a unitary hyperring.

For more details about the theory of hyperrings see [3, 4].

Throughout this paper R is a unitary hyperring and all related hypermodules are R–hypermodules.

Definition 1.3 (See [6]) A left *hypermodule* over a unitary hyperring R is a canonical hypergroup $(M, +)$ together with an external composition $. : R \times M \longrightarrow M$, denoted by $(r, m) \mapsto r.m \in M$, such that for all $x, y \in M$ and all $r, s \in R$, the following hold:

(M_1) : $r.(x + y) = r.x + r.y$;
(M_2) : $(r + s).x = r.x + s.x$;
(M_3) : $(rs).x = r.(s.x)$;
(M_4) : $1.m = m$ and $0.m = 0$, for each $m \in M$.

Let $(M, +)$ be an R–hypermodule and N be a nonempty subset of M. Then N is called a *subhypermodule* of M if $(N, +)$ is a canonical subhypergroup of $(M, +)$ and N is a hypermodule over R, under external composition . to $R \times N$. By $N \leq M$, we mean N is a subhypermodule of M.

Lemma 1.4 *Let M be a hypermodule and N be a nonvoid subset of M. Then N is a subhypermodule of M if and only if for every $x, y \in N$ and $r \in R$ we have $rx + y \subseteq N$.*

Proof. Obvious. □

Reader can refer to [2] for more information about hypermodules and subhypermodules and also about some special subhypermodules.

Let M, N be two R–hypermodules. A hyperoperation $f : M \longrightarrow N$ is called a *homomorphism* if for every pair $x, y \in M$ and every $r \in R$ the following hold

1. $f(x + y) = f(x) + f(y)$;
2. $f(rx) = rf(x)$,

and f is called a *weak homomorphism* if

1. $f(x + y) \subseteq f(x) + f(y)$;
2. $f(rx) = rf(x)$.

Note. For two hypermodules M, N and a homomorphism $f : M \longrightarrow N$, it is easy to see that $f(0) = 0$.

Let M be a hypermodule over a hyperring R and $N \leq M$. Consider $M/N = \{m+N \mid m \in M\}$, then M/N becomes a hypermodule over R under hyperoperation defined by $+ : M/N \times M/N \longrightarrow P^*(M/N)$ and external composition $. : R \times M/N \longrightarrow M/N$ such that $m+N+m'+N = \{x+N \mid x \in m+m'\}$ and $r.(m+N) = rm + N$ for $m, m' \in M$ and $r \in R$. Note that $m + N = N$ if and only if $m \in N$.

For a hypermodule M and a subhypermodule N of M there exists an epimorphism say *natural epimorphism* $\pi : M \longrightarrow M/N$ defined by $\pi(m) = m + N$ and obviously $Ker(\pi) = N$.

Note. ([6, corollary 3.2]) Let M, N be R–hypermodules. If $f : M \longrightarrow N$ is a homomorphism and $K \leq M$. Then

1. if $K \subseteq Ker(f)$, then there exists a unique homomorphism $\bar{f} : M/K \longrightarrow N$ such that $\bar{f}(m + K) = f(m)$ for every $m \in M$;

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2. if f is onto, then $\bar{f}$ is onto;
3. if $K = Ker(f)$, then $\bar{f}$ is one to one;
4. if f is onto and $K = Ker(f)$, then $\bar{f}$ is an isomorphism.

Let M be a hypermodule and A , B two subhypermodules of M. Define

$$A + B = \bigcup \{a + b | a \in A, b \in B\}$$

Then it is clear that $A + B$ is a subhypermodule of M.

Let M be a hypermodule and $A \leq M$, $B \leq M$; we have the following properties:
(i) $A + B = B$ if and only if $A \subseteq B$.
(ii) $A + \{0\} = A$.
(iii) If $C, D \leq A$ and $C, D \leq B$, then $C + D \subseteq A \cap B$.
(iv) If $a \in A$ and $b \in B$, then $a + b \subseteq A + B$.
Other trivial properties of sum of submodules which are satisfied in modules theory, are true also in hypermodules theory.

Remark. Let M be a hypermodule, N a subhypermodule of M and $x, y \in M$; then by properties of hypermodules we have
$x + N = y + N$ iff $N = (x + N) - (y + N) = (x - y + N) = \{t + N | t \in x - y\}$. So $x + N = y + N$ iff $N = t + N$ for some $t \in x - y$.
Also we have $(x + N) + (y + N) = N$ iff $x + y + N = N$ iff $x + y \subseteq N$.

Lemma 1.5 *Let M, N be hypermodules and K a subhypermodule of both of them. Then $M/K = N/K$ if and only if $M = N$.*

Proof. If $M = N$, then trivially $M/K = N/K$.
We prove the converse. Suppose that $M/K = N/K$ and $m \in M$. Then $m + K \in M/K = N/K$ and so there exists an element $n \in N$ such that $m + K = n + K$. Now by above Remark, $K = t + K$ for some $t \in m - n$ and so $t \in K$. Since $t \in m - n$, we have $m \in t + n$. Now $t \in K \subseteq N$ and $n \in N$. Therefore $m \in t + n \subseteq N$; i.e. $M \subseteq N$. By a similar way we obtain $N \subseteq M$. Thus $M = N$. $\square$

Let M be a hypermodule and $X \leq Y \leq M$, $L \leq M$. It is not difficult to see that $\frac{Y}{X} + \frac{L+X}{X} = \frac{L+Y}{X}$. In particular if A, B, C are subhypermodules of M such that $A + B = M$, then $\frac{A+C}{C} + \frac{B+C}{C} = \frac{M}{C}$.

Note. Let M, N be hypermodules and A, B be subhypermodules of M, N, respectively. If $f : M \longrightarrow N$ is a homomorphism, then it is clear to see that

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$f(A) = \{f(a)|a \in A\}$ is a subhypermodule of N and $f^{-1}(B) = \{x \in M|f(x) \in B\}$ is a subhypermodule of M.

Proposition 1.6 *Let M, N be hypermodules and $f : M \longrightarrow N$ a homomorphism. For two subhypermodules A, B of M we have the following statements*

1. *$f(a + b) \subseteq f(A + B)$ for every $a \in A$ and $b \in B$.*
2. *$f(A + B) = f(A) + f(B)$.*
3. *$Ker(f) = \{m \in M|f(m) = 0\}$ is a subhypermodule of M.*
4. *$Im(f) = \{f(m)|m \in M\}$ is a subhypermodule of N.*

Proof.
1. Since $a + b \subseteq A + B$, simply we can conclude $f(a + b) \subseteq f(A + B)$.
2. It is clear that $f(A) \subseteq f(A+B)$ and $f(B) \subseteq f(A+B)$, so $f(A)+f(B) \subseteq f(A+B)$.

Now let $x \in f(A + B)$. Then there exists an element $t \in A + B$ such that $x = f(t)$. So there exist $a \in A$ and $b \in B$ such that $t \in a + b$ and hence $x = f(t) \in f(a + b) = f(a) + f(b) \subseteq f(A) + f(B)$. This complete the proof.
Numbers 3 and 4 follows immediately from last note. □

2 Small subhypermodules

In this section we introduce a class of subhypermodules and proceed to get some suitable results about this kind of hypermodules.

Definition 2.1 Let M be a hypermodule and $N \leq M$, then N is called a *small subhypermodule* of M (denoted by $N \ll M$) if $N + K \neq M$ for all proper subhypermodule K of M; or equivalently $N + K = M$ implies $K = M$ for every $K \leq M$.

For two hypermodules M, N, an epimorphism $f : M \longrightarrow N$ is called a *small epimorphism* if $Ker(f) \ll M$.

Example 2.2
1. Consider the hypermodule $\frac{\mathbb{Z}}{4\mathbb{Z}}$ on hyperring $\mathbb{Z}$ with trivial hyperoperations. Then $\frac{2\mathbb{Z}}{4\mathbb{Z}} \ll \frac{\mathbb{Z}}{4\mathbb{Z}}$.
2. The hypermodule $\mathbb{Z}$ with trivial hyperoperations on $\mathbb{Z}$ has no small subhypermodules, because for every subhypermodule $n\mathbb{Z}$ of $\mathbb{Z}$, there exists a subhypermodule $m\mathbb{Z} \neq \mathbb{Z}$ of $\mathbb{Z}$ such that $n\mathbb{Z} + m\mathbb{Z} = \mathbb{Z}$; by getting a natural number $m \neq 1$ such that $(m, n) = 1$.

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3. Consider the hypermodule $\frac{\mathbb{Z}}{12\mathbb{Z}}$ with trivial hyperoperations on hyperring $\mathbb{Z}$. We have $\frac{3\mathbb{Z}}{12\mathbb{Z}} + \frac{4\mathbb{Z}}{12\mathbb{Z}} = \frac{\mathbb{Z}}{12\mathbb{Z}}$. So neither $\frac{3\mathbb{Z}}{12\mathbb{Z}}$ nor $\frac{4\mathbb{Z}}{12\mathbb{Z}}$ are small in $\frac{\mathbb{Z}}{12\mathbb{Z}}$. But it is not difficult to see that $\frac{6\mathbb{Z}}{12\mathbb{Z}} \ll \frac{\mathbb{Z}}{12\mathbb{Z}}$.

Proposition 2.3 *Let M be a hypermodule and K a subhypermodule of M. Then the following statements are equivalent*

1. *$K \ll M$;*

2. *The natural epimorphism $\pi : M \longrightarrow M/K$ is a small epimorphism;*

3. *For every hypermodule N and every homomorphism $f : N \longrightarrow M$,*

$$Im(f) + K = M \text{ implies } Im(f) = M.$$

Proof. Straightforward. □

Proposition 2.4 *Let M be a hypermodule and $X \leq Y$, N be subhypermodules of M. Then*
1. $Y \ll M$ if and only if $X \ll M$ and $Y/X \ll M/X$.
2. $N + Y \ll M$ if and only if $N \ll M$ and $Y \ll M$.

Proof. 1. Suppose that $Y \ll M$ and $X + L = M$ for some $L \leq M$. Since $X \leq Y$, we have $M = X + L \leq Y + L \leq M$. Hence $M = Y + L$ and so $L = M$ as $Y \ll M$. Now suppose that $Y/X + L/X = M/X$ for some $L \leq M$. Then $M = L + Y$ by Lemma 1.5. Thus $M = L$ and so $M/X = L/X$; i.e. $Y/X \ll M/X$.
For converse suppose that $X \ll M$ and $Y/X \ll M/X$. Let $M = Y + K$ for some $K \leq M$. Then

$$\frac{M}{X} = \frac{Y + K}{X} = \frac{Y}{X} + \frac{K + X}{X}.$$

Since $Y/X \ll M/X$, then $\frac{M}{X} = \frac{K+X}{X}$ and hence $M = K + X$ by Lemma 1.5. Now since $X \ll M$, we have $K = M$. This complete the proof.

2. Suppose that $N + Y \ll M$. Since $N \leq N + Y$ and $Y \leq N + Y$, simply we can conclude that $N \ll M$ and $Y \ll M$.
For converse suppose that $N \ll M$ and $Y \ll M$ and $M = L + N + Y$ for some $L \leq M$. By hypothesis we have $M = L + N$ and then $M = L$; i.e. $N + Y \ll M$. □

The following corollary is an immediate result from Proposition 2.4.

Corollary 2.5 *Let M be any hypermodule. Any finite sum of small subhypermodules of M is again small in M.*

Proposition 2.6 *Let M, N be hypermodules and K a subhypermodule of M. Moreover let $f : M \longrightarrow N$ be a homomorphism. If $K \ll M$, then $f(K) \ll N$.*

Proof. Suppose that $f(K) + L = N$ for some subhypermodule L of N. We first show that $K + f^{-1}(L) = M$. To see this, let $m \in M$. Then $f(m) \in N = f(K) + L$ and so there exist elements $k \in K$ and $l \in L$ such that $f(m) = f(k) + l$. Hence $l \in f(m) - f(k) = f(m - k)$. This causes the existence an element $t \in m - k$ such that $l = f(t)$. Since $t \in m - k$, so $m \in t + k = f^{-1}(l) + k \subseteq f^{-1}(L) + K$. Therefore $M \subseteq f^{-1}(L) + K$ and finally $M = f^{-1}(L) + K$. Now since $K \ll M$, we have $f^{-1}(L) = M$.
This implies $K \le f^{-1}(L)$ and then $f(K) \le L$. Now $N = f(K) + L = L$; i.e. $f(K) \ll N$. □

Corollary 2.7 *Let M be hypermodule and $K \le N \le M$ such that $K \ll N$. Then $K \ll M$.*

Proof. Consider the inclusion map $\iota : N \longrightarrow M$ and apply Proposition 2.6. □

Proposition 2.8 *Let M, N be hypermodules. Then an epimorphism $g : M \longrightarrow N$ is small if and only if for every homomorphism f, if gf is epimorphism, then f is epimorphism.*

Proof. Suppose that g is a small epimorphism; i.e. $Ker(g) \ll M$. Let L be a hypermodule and $f : L \longrightarrow M$ be a homomorphism such that gf is epic. First we show that $Im(f) + Ker(g) = M$. To see this let $m \in M$, then $g(m) \in N$. Since gf is epic, there exists an element $l \in L$ such that $g(m) = gf(l) = g(f(l))$. So $0 \in g(m) - g(f(l)) = g(m - f(l))$ and hence there exists an element $x \in m - f(l)$ such that $g(x) = 0$; i.e. $x \in Ker(g)$. Now we have $m \in x + f(l) \subseteq Ker(g) + Im(f)$ and consequently $M = Im(f) + Ker(g)$.
Now since $Ker(g) \ll M$, we have $Im(f) = M$; i.e. f is epic.
For converse let $Ker(g) + K = M$ for some subhypermodule K of M. Let $\iota : K \longrightarrow M$ be the inclusion map, then $g\iota : K \longrightarrow N$ is epic. Indeed let $n \in N$. Since g is epic, there exists $m \in M$ such that $n = g(m)$. Since $M = Ker(g) + K$, so there exist $x \in Ker(g)$ and $k \in K$ such that $m \in k + x$. Thus $g(m) \in g(k + x) = g(k) + g(x) = g(k) + 0 = \{g(k)\}$; i.e. $g(m) = g(k) = g(\iota(k) = g\iota(k)$. This implies that $g\iota$ is epic. Now by hypothesis ι must be epic and so $K = Im(\iota) = M$; i.e. $Ker(g) \ll M$. □

Definition 2.9 Let M be a hypermodule and N, K subhypermodules of M.

We say K and N are *independent*, if $K \cap N = 0$. If N, K are independent then $N + K$ is denoted by $N \oplus K$.

Also a subhypermodule N of M is called a *direct summand* of M if $M = N \oplus N'$ for some $N' \leq M$.

A hypermodule M is called *indecomposable* if whenever $M = M_1 \oplus M_2$, then $M_1 = 0$ or $M_2 = 0$.

Let M be any hypermodule and A, B and C be subhypermodules of M. Then it need not be that $A \cap (B + C) = (A \cap B) + (A \cap C)$.(see the following example)

Example 2.10 Let $M = \{(x, y)|x, y \in \mathbb{Z}\}$ with trivial hyperoperations on hyperring $\mathbb{Z}$. Also let

$$A = \{(x, x)|x \in \mathbb{Z}\}, B = \{(x, 0)|x \in \mathbb{Z}\} \textit{ and } C = \{(0, x)|x \in \mathbb{Z}\}.$$

Then A, B and C are subhypermodules of M and we have

$$A \cap (B + C) = A \neq 0 = (A \cap B) + (A \cap C).$$

In next proposition we add a condition that the above equality will be satisfied.

Lemma 2.11 *(modularity law) Suppose that M is a hypermodule and A, B, C are subhypermodules of M such that $B \leq A$. Then $A \cap (B + C) = B + (A \cap C)$.*

Proof. Clearly $B + (A \cap C) \subseteq A \cap (B + C)$.

Conversely let $x \in A \cap (B+C)$. Then $x = a \in b+c$ for some $a \in A, b \in B$ *and* $c \in C$. So we have $c \in a - b \subseteq A$, and hence $c \in A \cap C$. But $x \in b + c \subseteq B + A \cap C$. Thus $A \cap (B + C) \subseteq B + (A \cap C)$. □

Proposition 2.12 *Suppose that $M = M_1 \oplus M_2$ is a hypermodule where M_1, M_2 are subhypermodules of M. Then for each $m \in M$ there exist a unique element $m_1 \in M_1$ and a unique element $m_2 \in M_2$ such that $m \in m_1 + m_2$.*

Proof. Obviously, for each $m \in M$ there exist $m_1 \in M_1$ and $m_2 \in M_2$, such that $m \in m_1 + m_2$. Now suppose that $m \in m_1 + m_2$ and $m \in n_1 + n_2$ for some $m_1, n_1 \in M_1$ and $m_2, n_2 \in M_2$. Thus we have $0 \in m - m \subseteq (m_1 + m_2) - (n_1 + n_2) = (m_1 - n_1) + (m_2 - n_2)$ and so there exist $x \in m_1 - n_1 \subseteq M_1$ and $y \in m_2 - n_2 \subseteq M_2$ such that $0 \in x + y$. Hence $x = -y \in M_1 \cap M_2 =< 0 >$; i.e, $0 \in m_1 - n_1$ and $0 \in m_2 - n_2$ that shows $m_1 = n_1$ and $m_2 = n_2$. □

Proposition 2.13 *Let M be a hypermodule, N a direct summand of M and K a small subhypermodule of M contained in N. Then K is small in N.*

Proof. Suppose that $M = N \oplus N'$ for some $N' \leq M$. Also let $N = K + L$ for some $L \leq N$. Therefore $M = (K + L) \oplus N' = K + (L \oplus N')$. Since $K \ll M$, we conclude $M = L \oplus N'$. Now by modularity law we have $N = L + (N \cap N') = L + 0 = L$; i.e. $K \ll N$. □

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Proposition 2.14 *Let $K_1 \leq M_1 \leq M$ and $K_2 \leq M_2 \leq M$ be hypermodules such that $M = M_1 \oplus M_2$. Then*

$$K_1 \oplus K_2 \ll M_1 \oplus M_2 \quad iff \quad K_1 \ll M_1 \quad and \quad K_2 \ll M_2.$$

Proof. Suppose that $K_1 \ll M_1$ and $K_2 \ll M_2$, then by Corollary 2.7 we have $K_1 \ll M_1 \oplus M_2$ and also $K_2 \ll M_1 \oplus M_2$. Now by Proposition 2.4(ii), we deduce that $K_1 \oplus K_2 \ll M_1 \oplus M_2$.
For converse, suppose that $K_1 \oplus K_2 \ll M_1 \oplus M_2$. By Proposition 2.4 (i), we have $K_1 \ll M_1 \oplus M_2$ and $K_2 \ll M_1 \oplus M_2$. Now since $K_1 \leq M_1$ and $K_2 \leq M_2$, applying Proposition 2.13 the proof will be completed. □

Proposition 2.15 *Let M be a non-zero hypermodule and K be a small subhypermodule of M. If $\frac{M}{K}$ is indecomposable then so is M.*

Proof. Suppose that $M = M_1 \oplus M_2$. Then

$$\frac{M}{K} = \frac{M_1 + K}{K} \oplus \frac{M_2 + K}{K}.$$

Since M/K is indecomposable, either $\frac{M_1+K}{K} = \frac{M}{K}$ or $\frac{M_2+K}{K} = \frac{M}{K}$ and hence either $M_1+K = M$ or $M_2+K = M$. Now since $K \ll M$, we conclude that either $M_1 = M$ or $M_2 = M$, as required. □

Definition 2.16 Let M, N and K be hypermodules.
We say the sequence $0 \longrightarrow K \xrightarrow{f} M \xrightarrow{g} N \longrightarrow 0$ is an *exact sequence* if, f is a monomorphism, g is an epimorphism and $Im(f) = Ker(g)$.

Proposition 2.17 *Assume that the following diagram of hypermodules is commutative such that both rows are exact sequences and α is epic;*

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \xrightarrow{f} & B & \xrightarrow{g} & C & \longrightarrow & 0 \\ & & \downarrow\alpha & & \downarrow\beta & & \downarrow\gamma & & \\ 0 & \longrightarrow & A' & \xrightarrow{f'} & B' & \xrightarrow{g'} & C' & \longrightarrow & 0 \end{array}$$

If g is small, then so is g'.

Proof. Suppose that $Ker(g') + L' = B'$ for some $L' \leq B'$. Since α is epic, we have $(f'o\alpha)(A) = f'(\alpha(A)) = f'(A') = Im(f') = Ker(g')$. Now

$$Ker(g') = (f'o\alpha)(A) = (\beta of)(A) = \beta(f(A)) = \beta(Im(f)) = \beta(Ker(g)),$$

by the commutativity of diagram. So $\beta(Ker(g)) + L' = B'$. From the last statement we can show that $Ker(g) + \beta^{-1}(L') = \beta^{-1}(B') = B$. To see this let $x \in B$, then

$\beta(x) \in B' = \beta(Ker(g)) + L'$ and so there exist $y \in Ker(g)$ and $l' \in L'$ such that $\beta(x) \in \beta(y) + l'$. Therefore $l' \in \beta(x) - \beta(y) = \beta(x - y)$, and hence there exists an element $t \in x - y$ such that $l' = \beta(t)$. So $t = \beta^{-1}(l') \in \beta^{-1}(L')$. Now $x \in t + y \subseteq \beta^{-1}(L') + Ker(g)$. Hence $B \subseteq \beta^{-1}(L') + Ker(g)$. Also it is clear that $\beta^{-1}(L') + Ker(g) \subseteq B$. Since $Ker(g) \ll B$, we conclude that $B = \beta^{-1}(L')$ and hence $L' = B'$; that is $Ker(g') \ll B'$. □

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