

EDGE-TO-VERTEX DETOUR MONOPHONIC NUMBER OF A GRAPH

A.P. SANTHAKUMARAN, P. TITUS AND K. GANESAMOORTHY

ABSTRACT. For a connected graph $G = (V, E)$ of order at least three, the monophonic distance $d_m(u, v)$ is the length of a longest $u - v$ monophonic path in G . For subsets A and B of V , the monophonic distance $d_m(A, B)$ is defined as $d_m(A, B) = \min\{d_m(x, y) : x \in A, y \in B\}$. A $u - v$ path of length $d_m(A, B)$ is called an $A - B$ detour monophonic path joining the sets $A, B \subseteq V$, where $u \in A$ and $v \in B$. A set $S \subseteq E$ is called an *edge-to-vertex detour monophonic set* of G if every vertex of G is incident with an edge of S or lies on a detour monophonic joining a pair of edges of S . The *edge-to-vertex detour monophonic number* $dm_{ev}(G)$ of G is the minimum order of its edge-to-vertex detour monophonic sets and any edge-to-vertex detour monophonic set of order $dm_{ev}(G)$ is an *edge-to-vertex detour monophonic basis* of G . Certain general properties of these concepts are studied. It is shown that for each pair of integers k and q with $2 \leq k \leq q$, there exists a connected graph G of order $q + 1$ and size q with $dm_{ev}(G) = k$.

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1. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q , respectively. For basic graph theoretic terminology we refer to Harary [1, 5]. For vertices x and y in a connected graph G , the *distance* $d(x, y)$ is the length of a shortest $x - y$ path in G . An $x - y$ path of length $d(x, y)$ is called an $x - y$ *geodesic*. The *neighborhood* of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v . A vertex v is an *extreme vertex* if the subgraph induced by its neighbors is complete.

The *detour distance* $D(u, v)$ between two vertices u and v in G is the length of a longest $u - v$ path in G . An $u - v$ path of length $D(u, v)$ is called an $u - v$ *detour*. It is known that D is a metric on the vertex set V of G . The closed detour interval $I_D[x, y]$ consists of x, y , and all the vertices in some $x - y$ detour of G . For $S \subseteq V$, $I_D[S]$ is the union of the sets $I_D[x, y]$ for all $x, y \in S$. A set S of vertices is a *detour set* if $I_D[S] = V$, and the minimum cardinality of a detour set is the *detour number* $dn(G)$. The concept of detour number was introduced in [2, 3] and further studied in [3, 4].

A *chord* of a path P is an edge joining two non-adjacent vertices of P . A path P is called *monophonic* if it is a chordless path. A longest $x - y$ monophonic path is called an $x - y$ *detour monophonic path*. A set S of vertices of a graph G is a *detour monophonic*

set if each vertex v of G lies on an $x - y$ detour monophonic path for some $x, y \in S$. The minimum cardinality of a detour monophonic set of G is the *detour monophonic number* of G and is denoted by $dm(G)$. The detour monophonic number of a graph was introduced in [9] and further studied in [10].

An *edge detour monophonic set* of G is a set S of vertices such that every edge of G lies on a detour monophonic path joining some pair of vertices in S . The *edge detour monophonic number* of G is the minimum cardinality of its edge detour monophonic sets and is denoted by $edm(G)$. An edge detour monophonic set of cardinality $edm(G)$ is an *edm-set* of G . The edge detour monophonic number of a graph was introduced and studied in [8].

For any two vertices u and v in a connected graph G , the *monophonic distance* $d_m(u, v)$ from u to v is defined as the length of a longest $u - v$ monophonic path in G . The *monophonic eccentricity* $e_m(v)$ of a vertex v in G is $e_m(v) = \max \{d_m(v, u) : u \in V(G)\}$. The *monophonic radius*, $rad_m G$ of G is $rad_m(G) = \min \{e_m(v) : v \in V(G)\}$ and the *monophonic diameter*, $diam_m G$ of G is $diam_m(G) = \max \{e_m(v) : v \in V(G)\}$. A vertex u in G is a *monophonic eccentric vertex* of a vertex v in G if $e_m(v) = d_m(u, v)$. The monophonic distance was introduced in [6] and further studied in [7].

Throughout this paper G denotes a connected graph with at least three vertices.

2. EDGE-TO-VERTEX DETOUR MONOPHONIC NUMBER

Definition 2.1. Let $G = (V, E)$ be a connected graph with at least three vertices. For subsets A and B of V , the *monophonic distance* $d_m(A, B)$ is defined as $d_m(A, B) = \min\{d_m(x, y) : x \in A, y \in B\}$. A $u - v$ detour monophonic path of length $d_m(A, B)$ is called an *$A - B$ detour monophonic path* joining the sets A and B , where $u \in A$ and $v \in B$. For $A = \{u, v\}$ and $B = \{z, w\}$ with uv and zw edges, we write an $A - B$ detour monophonic path as $uv - zw$ detour monophonic path, and $d_m(A, B)$ as $d_m(uv, zw)$.

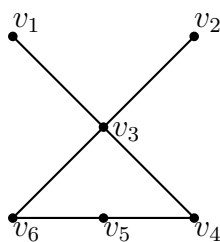


Figure 2.1: G

Example 2.2. For the graph G given in Figure 2.1, with $A = \{v_1, v_2\}$ and $B = \{v_4, v_5\}$, $P : v_1, v_3, v_4$ is the only $v_1 - v_4$ detour monophonic path; $Q : v_1, v_3, v_4, v_5$ and $R : v_1, v_3, v_6, v_5$ are the only $v_1 - v_5$ detour monophonic paths; $P' : v_2, v_3, v_4$ is the only $v_2 - v_4$ detour monophonic path, $Q' : v_2, v_3, v_4, v_5$ and $R' : v_2, v_3, v_6, v_5$ are the only $v_2 - v_5$ detour monophonic paths. Hence $d_m(A, B) = 2$ and $P : v_1, v_3, v_4$ and $P' : v_2, v_3, v_4$ are the only two $A - B$ detour monophonic paths.

Definition 2.3. Let $G = (V, E)$ be a connected graph with at least three vertices. A set $S \subseteq E$ is called an *edge-to-vertex detour monophonic set* of G if every vertex of G is incident with an edge of S or lies on a detour monophonic path joining a pair of edges of S . The *edge-to-vertex detour monophonic number* $dm_{ev}(G)$ of G is the minimum cardinality of its edge-to-vertex detour monophonic sets and any edge-to-vertex detour monophonic set of cardinality $dm_{ev}(G)$ is an *edge-to-vertex detour monophonic basis* of G .

Example 2.4. For the graph G given in Figure 2.2, the four $v_1v_2-v_4v_5$ detour monophonic paths are $P_1 : v_1, v_2, v_3, v_4$, $P_2 : v_1, v_6, v_5, v_4$, $Q_1 : v_2, v_3, v_4, v_5$ and $Q_2 : v_2, v_1, v_6, v_5$, each of length 3 so that $d_m(v_1v_2, v_4v_5) = 3$. Since the vertices v_3 and v_6 lie on the $v_1v_2 - v_4v_5$ detours monophonic paths P_1 and P_2 respectively, $S_1 = \{v_1v_2, v_4v_5\}$ is an edge-to-vertex detour monophonic basis of G so that $dm_{ev}(G) = 2$. Also $S_2 = \{v_2v_3, v_5v_6\}$ and $S_3 = \{v_3v_4, v_1v_6\}$ are edge-to-vertex detour monophonic bases of G . Thus there can be more than one edge-to-vertex detour monophonic basis for a graph.

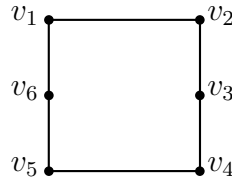


Figure 2.2: G

It is clear that an edge-to-vertex detour monophonic set needs at least two edges, and the set of all edges of G is an edge-to-vertex detour monophonic set of G . Hence the following proposition is trivial.

Proposition 2.5. For any connected graph G of size $q \geq 2$, $2 \leq dm_{ev}(G) \leq q$.

For the star $K_{1,q}(q \geq 2)$, it is clear that the set of all edges is the unique edge-to-vertex detour monophonic set so that $dm_{ev}(K_{1,q}) = q$. The set of two end-edges of a path $P_n(n \geq 3)$ is its unique edge-to-vertex detour monophonic basis so that $dm_{ev}(P_n) = 2$. Thus the bounds in Proposition 2.5 are sharp.

Definition 2.6. An edge e in a graph G is an *edge-to-vertex detour monophonic edge* in G if e belongs to every edge-to-vertex detour monophonic basis of G . If G has a unique edge-to-vertex detour monophonic basis S , then every edge in S is an edge-to-vertex detour monophonic edge of G .

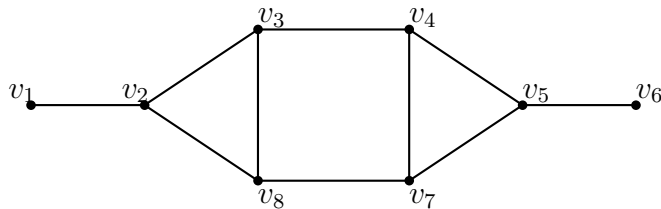


Figure 2.3: G

Example 2.7. For the graph G given in Figure 2.3, $S = \{v_1v_2, v_5v_6\}$ is the unique edge-to-vertex detour monophonic basis of G so that both the edges in S are edge-to-vertex detour monophonic edge of G . For the graph G given in Figure 2.1, it is easily verified that no two element subset of E is an edge-to-vertex detour monophonic set of G . Also, it is clear that $S_1 = \{v_1v_3, v_2v_3, v_4v_5\}$ and $S_2 = \{v_1v_3, v_2v_3, v_5v_6\}$ are the only edge-to-vertex detour monophonic bases of G so that the edges v_1v_3, v_2v_3 are the edge-to-vertex detour monophonic edges of G .

An edge of a connected graph G is called an *extreme edge* of G if one of its ends is an extreme vertex of G .

Theorem 2.8. *If v is an extreme vertex of a non-complete connected graph G , then every edge-to-vertex detour monophonic set of G contains at least one extreme edge that is incident with v .*

Proof. Let v be an extreme vertex of G . Let e_1, e_2, \dots, e_k be the edges incident with v . Let S be any edge-to-vertex detour monophonic set of G . We claim that $e_i \in S$ for some $i(1 \leq i \leq k)$. Otherwise, $e_i \notin S$ for any $i(1 \leq i \leq k)$. Since S is an edge-to-vertex detour monophonic set and the vertex v is not incident with any element of S , v lies on a detour monophonic path joining two elements say $x, y \in S$. Let $x = v_1v_2$ and $y = v_l v_m$. Then $v \neq v_1, v_2, v_l, v_m$ and since G is non-complete, $d_m(x, y) \geq 2$. Let u and w be the neighbors of v on P . Then u and w are not adjacent and so v is not an extreme vertex, which is a contradiction. Therefore, $e_i \in S$ for some $i(1 \leq i \leq k)$. \square

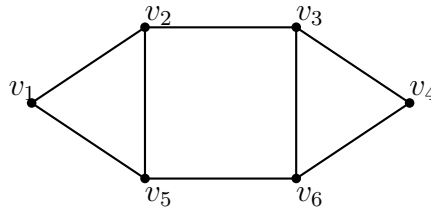


Figure 2.4: G

Remark 2.9. For the graph G given in Figure 2.4, $S = \{v_1v_5, v_3v_4\}$ is an edge-to-vertex detour monophonic set of G , which does not contain the extreme edge v_1v_2 . Thus all the extreme edges of a graph need not belong to an edge-to-vertex detour monophonic set of G .

In the following theorem we show that there are certain edges in a connected graph G that are edge-to-vertex detour monophonic edges of G .

Corollary 2.10. *Every end-edge of a connected graph G belongs to every edge-to-vertex detour monophonic set of G . Also if the set S of all end-edges of G is an edge-to-vertex detour monophonic set, then S is the unique edge-to-vertex detour monophonic basis for G .*

Proof. This follows from Theorem 2.8. If S is the set of all end-edges of G , then by the first part of this corollary $dm_{ev}(G) \geq |S|$. Since S is an edge-to-vertex detour monophonic

set of G , $dm_{ev}(G) \leq |S|$. Hence $dm_{ev}(G) = |S|$ and S is the unique edge-to-vertex detour monophonic basis for G . \square

Corollary 2.11. *If T is a tree with k end-edges, then $dm_{ev}(T) = k$.*

Corollary 2.12. *For any connected graph G with k end-edges, $\max\{2, k\} \leq dm_{ev}(G) \leq q$.*

Proof. This follows from Proposition 2.5 and Corollary 2.10. \square

For a cutvertex v in a connected graph G and a component H of $G - v$, the subgraph H and the vertex v together with all edges joining v and $V(H)$ is called a *branch* of G at v .

Theorem 2.13. *Let G be a connected graph with cutvertices and S an edge-to-vertex detour monophonic set of G . Then every branch of G contains an element of S .*

Proof. Assume that there is a branch B of G at a cutvertex v such that B contains no element of S . Then by Corollary 2.10, B does not contain any end-edge of G . Hence it follows that no vertex of B is an endvertex of G . Let u be any vertex of B (note that $|V(B)| \geq 2$). Then u is not incident with any end-edge of G and so u lies on a $e - f$ detour monophonic path $P : u_1, u_2, \dots, u, \dots, u_t$ where u_1 is an end of e , u_t is an end of f and $e, f \in S$. Since v is a cutvertex of G , the $u_1 - u$ and $u - u_t$ subpaths of P both contain v and so P is not a path, which is a contradiction. Hence every branch of G contains an element of S . \square

Corollary 2.14. *Let G be a connected graph with cut-edges and S an edge-to-vertex detour monophonic set of G . Then every branch of G contains an element of S .*

Corollary 2.15. *Let G be a connected graph with cut-edges and S an edge-to-vertex detour monophonic set of G . Then for any cut-edge e of G , which is not an end-edge, each component of $G - e$ contains an element of S .*

Proof. Let $e = uv$. Let G_1 and G_2 be the two components of $G - e$ such that $u \in V(G_1)$ and $v \in V(G_2)$. Since u and v are cutvertices of G , the result follows from Theorem 2.13. \square

Corollary 2.16. *If G is a connected graph with $k \geq 2$ endblocks, then $dm_{ev}(G) \geq k$.*

Corollary 2.17. *If G is a connected graph with a cutvertex v and the number of components of $G - v$ is r , then $dm_{ev}(G) \geq r$.*

Remark 2.18. By Corollary 2.16, if S is an edge-to-vertex detour monophonic set of a graph G , then every endblock of G must contain at least one element of S . However, it is possible that some blocks of G that are not endblocks must contain an element of S as well. For example, consider the graph G given in Figure 2.5, where the cycle $C_5 : x, y, t, w, s, x$ is a block of G that is not an endblock. By Corollary 2.10, every edge-to-vertex detour monophonic set of G must contain us and tv . Since the $us - tv$ detour monophonic path does not contain the vertex w , it follows that $\{us, tv\}$ is not an edge-to-vertex detour monophonic set of G . Thus every edge-to-vertex detour monophonic set of G must contain at least one edge from the block C_5 .

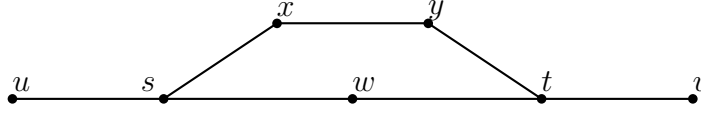


Figure 2.5: G

Theorem 2.19. *Let G be a connected graph with cut-edges. Then no cut-edge which is not an end-edge in G belongs to any edge-to-vertex detour monophonic basis of G .*

Proof. Suppose that S is an edge-to-vertex detour monophonic basis that contains a cut-edge $e = uv$ which is not an end-edge of G . Let G_1, G_2 be the two components of $G - e$ such that $u \in G_1$ and $v \in G_2$. Then by Corollary 2.15, each of G_1 and G_2 contains an element of S . Let $S' = S - \{uv\}$. We show that S' is an edge-to-vertex detour monophonic set of G . Since S is an edge-to-vertex detour monophonic set of G and $uv \in S$, let s be any vertex of G that lies on a detour monophonic path P joining the edges, say xy and uv of S . We may assume that $xy \in E(G_1)$ and so $V(P) \subseteq V(G_1)$. Let P_1 be the $xy - uv$ detour monophonic path that contains the vertex s and P_2 be any $uv - wz$ detour monophonic path in G , where $wz \in E(G_2) \cap S$. Then, since uv is a cut-edge of G , the detour monophonic path P_1 followed by the edge uv and the detour monophonic path P_2 is an $xy - wz$ detour monophonic path which contains the vertex s . Thus we have shown that a vertex that lies on a detour monophonic path joining a pair of edges xy and uv of S also lies on a detour monophonic path joining a pair of edges xy and wz of S' . Hence it follows that S' is an edge-to-vertex detour monophonic set of G . Since $|S'| = |S| - 1$, this contradicts that S is an edge-to-vertex detour monophonic basis of G . Thus the result is proved. \square

3. EDGE-TO-VERTEX DETOUR MONOPHONIC NUMBERS OF SOME STANDARD GRAPHS

Theorem 3.1. *For p even, a set S of edges of $G = K_p (p \geq 4)$ is an edge-to-vertex detour monophonic basis of K_p if and only if S consists of $p/2$ independent edges.*

Proof. Let S be any set of $p/2$ independent edges of K_p . Since each vertex of K_p is incident with an edge of S , it follows that $dm_{ev}(G) \leq p/2$. If $dm_{ev}(G) < p/2$, then there exists an edge-to-vertex detour monophonic set S' of K_p such that $|S'| < p/2$. Therefore, there exists at least one vertex v of K_p such that v is not incident with any edge of S' . Since $d_m(e, f) = 1$ if e and f are independent edges, it follows that v is neither incident with any edge of S' nor lies on a detour monophonic path joining a pair of edges of S' and so S' is not an edge-to-vertex detour monophonic set of G , which is a contradiction. Thus S is an edge-to-vertex detour monophonic basis of K_p .

Conversely, let S be an edge-to-vertex detour monophonic basis of K_p . Let S' be any set of $p/2$ independent edges of K_p . Then as in the first part of this theorem, S' is an edge-to-vertex detour monophonic basis of K_p . Therefore, $|S| = p/2$. If S is not independent, then there exists a vertex v of K_p such that v is not incident with any edge

of S and it follows that S is not an edge-to-vertex detour monophonic set of G , which is a contradiction. Therefore, S consists of $p/2$ independent edges. \square

Corollary 3.2. *For the complete graph $K_p(p \geq 4)$ with p even, $dm_{ev}(K_p) = p/2$.*

For any real x , $\lceil x \rceil$ denotes the smallest integer greater than or equal to x .

Theorem 3.3. *For the complete graph $G = K_p(p \geq 3)$ with p odd, $dm_{ev}(G) = \frac{p+1}{2}$.*

Proof. Let S be any set of $\frac{p-1}{2}$ independent edges of G . Then there exists a unique vertex v which is not incident with an edge of S . Let S_1 be the union of S and an edge incident with v . Then S_1 is an edge-to-vertex detour monophonic set of G so that $dm_{ev}(G) \leq \frac{p-1}{2} + 1$. Now, if $dm_{ev}(G) \leq \frac{p-1}{2}$, then let S_2 be an edge-to-vertex detour monophonic set of G such that $|S_2| \leq \frac{p-1}{2}$. Then there exists a vertex u such that u is not incident with any edge of S_2 . Obviously, u does not lie on a detour monophonic path joining a pair of edges of S_2 , which is a contradiction to S_2 an edge-to-vertex detour monophonic set of G . Hence $dm_{ev}(G) = \frac{p-1}{2} + 1 = \frac{p+1}{2}$. \square

Corollary 3.4. *For the complete graph $K_p(p \geq 3)$, $dm_{ev}(K_p) = \lceil \frac{p}{2} \rceil$.*

Two vertices u and v of G are called *antipodal* if $d(u, v) = \text{diam } G$, where $\text{diam } G$ is the usual diameter of the graph G .

Theorem 3.5. *For the cycle $C_p(p \geq 3)$, $dm_{ev}(C_p) = \begin{cases} 2 & \text{if } p \neq 5 \\ 3 & \text{if } p = 5. \end{cases}$*

Proof. For $p = 3$, $C_p = K_3$ and any set of two edges is an edge-to-vertex detour monophonic basis and so $dm_{ev}(G) = 2$.

Let $p \geq 4$ and $p \neq 5$. Let $C_p : v_1, v_2, v_3, \dots, v_k, v_{k+1}, v_{k+2}, \dots, v_p, v_1$ be the cycle of order p such that v_{k+1} is the unique antipodal vertex of v_1 if p is even; and v_{k+1} and v_{k+2} are the antipodal vertices of v_1 if p is odd. Then it is easily checked that $S = \{v_1v_2, v_{k+1}v_{k+2}\}$ is an edge-to-vertex detour monophonic set of C_p so that $dm_{ev}(C_p) = 2$.

For $p = 5$, it is easily seen that no 2-element subset of edges of C_5 is an edge-to-vertex detour monophonic set of C_5 since $d_m(e, f) = 1$ if e and f are two independent edges in C_5 . Also, since $S = \{v_1v_2, v_2v_3, v_4v_5\}$ is an edge-to-vertex detour monophonic set of C_5 , it follows that $dm_{ev}(C_5) = 3$. \square

4. MONOPHONIC DIAMETER AND EDGE-TO-VERTEX DETOUR MONOPHONIC NUMBER

Theorem 4.1. *For each pair of integers k and q with $2 \leq k \leq q$, there exists a connected graph G of order $q + 1$ and size q with $dm_{ev}(G) = k$.*

Proof. For $2 \leq k \leq q$, let P be a path of order $q - k + 3$. Then the graph G obtained from P by adding $k - 2$ new vertices to P and joining them to any cutvertex of P is a tree of order $q + 1$ and size q with k end-edges and so by Corollary 2.11, $dm_{ev}(G) = k$. \square

Proposition 2.5 shows that if G is a connected graph of size $q \geq 2$, then $2 \leq dm_{ev}(G) \leq q$. Indeed, by Theorem 4.1, for each pair k, q of integers with $2 \leq k \leq q$, there is a tree of size q with edge-to-vertex detour monophonic number k . An improved upper bound for the edge-to-vertex detour monophonic number of a graph can be given in terms of its size q and detour monophonic diameter. For convenience, we denote the detour monophonic diameter $diam_m(G)$ by d_m itself.

Theorem 4.2. *If G is a connected graph of size q and monophonic diameter d_m , then $dm_{ev}(G) \leq q - d_m + 2$.*

Proof. Let u and v be vertices of G such that $d_m(u, v) = d_m$ and let $P : u = v_0, v_1, v_2, \dots, v_{d_m-1}, v_{d_m} = v$ be a $u - v$ detour monophonic path of length d_m . Let $S = (E(G) - E(P)) \cup \{uv_1, v_{d_m-1}v\}$. Then it is clear that S is an edge-to-vertex detour monophonic set of G so that $dm_{ev}(G) \leq |S| = q - d_m + 2$. \square

We give below a characterization theorem for trees.

Theorem 4.3. *For any tree T of size $q \geq 2$ and monophonic diameter d_m , $dm_{ev}(T) = q - d_m + 2$ if and only if T is a caterpillar.*

Proof. Let T be any tree of size $q \geq 2$ and $P : v_0, v_1, \dots, v_{d_m-1}, v_{d_m}$ be a monophonic diametral path of T . Let $e_1, e_2, \dots, e_{d_m-1}, e_{d_m}$ be the edges of P , where $e_i = v_{i-1}v_i$ ($1 \leq i \leq d_m$), k the number of end-edges of T and l the number of internal edges of T other than e_2, \dots, e_{d_m-1} . Then $k + l + d_m - 2 = q$. By Corollary 2.11, $dm_{ev}(T) = k = q - d_m - l + 2$. Hence $dm_{ev}(T) = k = q - d_m + 2$ if and only if $l = 0$, if and only if all the internal edges of T lie on the monophonic diametral path P , if and only if T is a caterpillar. \square

Corollary 4.4. *For a wounded spider T of size $q \geq 2$, $dm_{ev}(T) = q - d_m + 2$ if and only if T is obtained from $K_{1,t}$ ($t \geq 2$) by subdividing at most two of its edges.*

Proof. Since a wounded spider T is a caterpillar if and only if T is obtained from $K_{1,t}$ ($t \geq 2$) by subdividing at most two of its edges, the result follows from Theorem 4.3. \square

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DEPARTMENT OF MATHEMATICS, HINDUSTAN UNIVERSITY, HINDUSTAN INSTITUTE OF TECHNOLOGY AND SCIENCE, CHENNAI - 603 103, INDIA
E-mail address: apskumar1953@yahoo.co.in

DEPARTMENT OF MATHEMATICS, UNIVERSITY COLLEGE OF ENGINEERING NAGERCOIL, ANNA UNIVERSITY, TIRUNELVELI REGION, NAGERCOIL - 629 004, INDIA
E-mail address: titusvino@yahoo.com

DEPARTMENT OF MATHEMATICS, UNIVERSITY V.O.C. COLLEGE OF ENGINEERING, ANNA UNIVERSITY, TIRUNELVELI REGION, TUTICORIN - 628 008, INDIA
E-mail address: kvgm_2005@yahoo.co.in