

ON NEW CESÀRO-ORLICZ DOUBLE DIFFERENCE SEQUENCE SPACE

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ABSTRACT. The aim of this paper is to introduce the Cesàro-Orlicz double difference sequence space $Ces_M^{(2)}(\Delta, p)$. We study some topological properties of this space and give some inclusion relations.

1. INTRODUCTION

Throughout this work, \mathbb{N} , \mathbb{R} , w and w^2 denote the sets of positive integers, real numbers, single real sequences and double real sequences, respectively.

First of all, let us recall preliminary definitions and notations.

A double sequence on a normed linear space X is a function x from $\mathbb{N} \times \mathbb{N}$ into X and briefly denoted by $x = (x_{kl})$. If for every $\varepsilon > 0$ there exists $n_\varepsilon \in \mathbb{N}$ such that $\|x_{kl} - a\|_X < \varepsilon$ whenever $k, l > n_\varepsilon$ then a double sequence (x_{kl}) is said to be converges (in terms of Pringsheim) to $a \in X$ [16].

A double series $\sum_{k,l=1}^{\infty} x_{kl}$ is convergent if and only if its sequence of partial sums (s_{nm}) is convergent (see [2],[3]), where $s_{nm} = \sum_{k=1}^n \sum_{l=1}^m x_{kl}$ for all $m, n \in \mathbb{N}$.

Let X be a linear space. A function $p : X \rightarrow \mathbb{R}$ is called *paranorm*, if

i) $p(0) = 0$,

ii) $p(x) \geq 0$ for all $x \in X$,

iii) $p(-x) = p(x)$ for all $x \in X$,

iv) $p(x + y) \leq p(x) + p(y)$ for all $x, y \in X$,

v) if (λ_n) is a sequence of scalars with $\lambda_n \rightarrow \lambda$ as $n \rightarrow \infty$ and (x_n) is a sequence of vectors with $p(x_n - x) \rightarrow 0$ as $n \rightarrow \infty$, then $p(\lambda_n x_n - \lambda x) \rightarrow 0$ as $n \rightarrow \infty$ (continuity of scalars multiplication).

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called *total* [12].

An Orlicz function is a function $M : [0, \infty) \rightarrow [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$.

An Orlicz function M can always be represented in the following integral form:

$M(x) = \int_0^x \eta(t) dt$, where η is known as the kernel of M , is right differentiable for $t \geq 0$, $\eta(0) = 0$, $\eta(t) > 0$ for $t > 0$, η is nondecreasing and $\eta(t) \rightarrow \infty$ as $t \rightarrow \infty$.

An Orlicz function M is said to be satisfied the Δ_2 -condition if there are $T > 0$ and $a > 0$ such that $M(a) > 0$ and $M(2u) \leq TM(u)$ for all $u \in [0, a]$ (see [10]).

For $1 \leq p < \infty$, the Cesàro sequence space Ces_p is defined by

2000 *Mathematics Subject Classification.* 40A05, 40A25, 40A30.

Key words and phrases. Double sequences, Orlicz function, paranorm, Cesàro sequence space.

$$Ces_p = \left\{ x \in w : \sum_{j=1}^{\infty} \left(\frac{1}{j} \sum_{i=1}^j |x_i| \right)^p < \infty \right\},$$

equipped with norm

$$\|x\| = \left(\sum_{j=1}^{\infty} \left(\frac{1}{j} \sum_{i=1}^j |x_i| \right)^p \right)^{\frac{1}{p}}.$$

This space was first introduced by Shiue [18]. It is very useful in the theory of matrix operators and others. Sanhan and Suantai introduced and studied a generalized Cesàro sequence space $Ces(p)$, where $p = (p_j)$ is a bounded sequence of positive real numbers (see [17]). Later, this space was studied by many authors in [4], [7], [9], [11], [14], [15].

The notion of difference sequence space was introduced by Kızmaz in [8] in 1981 as follows:

$$X(\Delta) = \{x = (x_k) \in w : (x_k - x_{k+1}) \in X\}$$

for $X = \ell_{\infty}, c, c_0$. Subsequently difference sequence spaces has been discussed in Ahmad and Mursaleen [1], Malkowsky and Parashar [13], Et and Başarır [5], Et and Çolak [6] and others.

In this work, we introduce double sequence spaces $Ces_M^{(2)}(\Delta, p)$ as follows;

Let $p = (p_{nm})$ be a bounded double sequence of positive real numbers and M be an Orlicz function. The space $Ces_M^{(2)}(\Delta, p)$ is defined by

$$Ces_M^{(2)}(\Delta, p) = \left\{ x \in w^2 : \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty, \exists \rho > 0 \right\},$$

where $\Delta x_{ij} = x_{i-1,j-1} - x_{i-1,j} - x_{i,j-1} + x_{ij}$ for all $i, j \in \mathbb{N}$ and the terms with negative subscript are assume zero.

The following inequality will be used throughout this paper. Let (p_{nm}) be a bounded double sequence of strictly positive real numbers and denote $H = \sup_{n,m} p_{nm}$. For any complex a_{nm} and b_{nm} we have

$$|a_{nm} + b_{nm}|^{p_{nm}} \leq D. (|a_{nm}|^{p_{nm}} + |b_{nm}|^{p_{nm}})$$

where $D = \max(1, 2^{H-1})$. Also, for any complex λ ,

$$|\lambda|^{p_{nm}} \leq \max(1, |\lambda|^H).$$

2. MAIN RESULTS

Theorem 1. *Let (p_{nm}) be bounded. The set $Ces_M^{(2)}(\Delta, p)$ of double sequences is a linear space over the real field \mathbb{R} .*

Proof. Let $x, y \in Ces_M^{(2)}(\Delta, p)$ and $\lambda, \beta \in \mathbb{C}$. Then there exist $\rho_1 > 0, \rho_2 > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} < \infty$$

and

$$\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} < \infty.$$

Let $\alpha, \beta \in \mathbb{R}$ and $\rho_3 = \max \{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since M is non-decreasing convex function, we have

$$\begin{aligned} & \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\alpha\Delta x_{ij} + \beta\Delta y_{ij}|}{\rho_3} \right) \right]^{p_{nm}} \\ & \leq \sum_{n,m=1}^{\infty} \left[M \left(\frac{|\alpha|}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_3} + \frac{|\beta|}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_3} \right) \right]^{p_{nm}} \\ & \leq \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{2nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} + \frac{1}{2nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \\ & \leq \sum_{n,m=1}^{\infty} \frac{1}{2^{p_{nm}}} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) + M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \\ & < \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) + M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \\ & \leq D \cdot \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} \\ & \quad + D \cdot \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \\ & < \infty, \end{aligned}$$

where $D = \max(1, 2^{H-1})$. This shows that $\lambda x + \beta y \in Ces_M^{(2)}(\Delta, p)$ and so $Ces_M^{(2)}(\Delta, p)$ is a linear space. \square

Theorem 2. *The double sequence space $Ces_M^{(2)}(\Delta, p)$ is a paranormed space with the paranorm*

$$g(x) = \inf \left\{ \rho^{\frac{pqr}{R}} > 0 : \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \leq 1, q, r \in \mathbb{N} \right\}$$

where $H = \sup_{n,m} p_{nm} < \infty$ and $R = \max(1, H)$.

Proof. It is clear that $g(x) = g(-x)$ and $g(0) = 0$. For any $x, y \in Ces_M^{(2)}(\Delta, p)$, there exist $\rho_1, \rho_2 > 0$ such that

$$\left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \leq 1$$

and

$$\left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \leq 1.$$

Let $\rho_3 = 2^{\frac{R}{h}}(\rho_1 + \rho_2)$, where $h = \inf p_{nm} > 0$. Since M is a non-decreasing convex function, we have

$$\begin{aligned} & \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij} + \Delta y_{ij}|}{\rho_3} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \leq \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{2^{\frac{R}{h}}(\rho_1 + \rho_2)} + \frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{2^{\frac{R}{h}}(\rho_1 + \rho_2)} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \leq \left(\sum_{n,m=1}^{\infty} \left[\frac{\rho_1}{2^{\frac{R}{h}}(\rho_1 + \rho_2)} M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right. \right. \\ & \quad \left. \left. + \frac{\rho_2}{2^{\frac{R}{h}}(\rho_1 + \rho_2)} M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \leq \left(\sum_{n,m=1}^{\infty} \left[\frac{1}{2^{\frac{R}{h}}} M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \quad + \left(\sum_{n,m=1}^{\infty} \left[\frac{1}{2^{\frac{R}{h}}} M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & = \frac{1}{2} \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \quad + \frac{1}{2} \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \\ & \leq 1. \end{aligned}$$

Since ρ_1, ρ_2, ρ_3 are positive real numbers we get

$$g(x+y) = \inf \left\{ \rho_3^{\frac{p_{qr}}{R}} > 0 : \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij} + \Delta y_{ij}|}{\rho_3} \right) \right]^{p_{nm}} \right)^{\frac{1}{R}} \leq 1; q, r \in \mathbb{N} \right\}$$

$$\begin{aligned}
&\leq \inf \left\{ \rho_1^{\frac{pqr}{R}} > 0 : \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho_1} \right) \right]^{pnm} \right)^{\frac{1}{R}} \leq 1; q, r \in \mathbb{N} \right\} \\
&\quad + \inf \left\{ \rho_2^{\frac{pqr}{R}} > 0 : \left(\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta y_{ij}|}{\rho_2} \right) \right]^{pnm} \right)^{\frac{1}{R}} \leq 1; q, r \in \mathbb{N} \right\} \\
&= g(x) + g(y).
\end{aligned}$$

Let $(x^n) = \{x_{ij}^n\}$ be any sequence in the space $Ces_M^{(2)}(\Delta, p)$ such that $g(x^n - x) \rightarrow 0$, as $n \rightarrow \infty$ and (λ_n) is a sequence of reals with $\lambda_n \rightarrow \lambda$, as $n \rightarrow \infty$. Then, since the inequality

$$g(x^n) \leq g(x) + g(x^n - x)$$

holds by subadditivity of the function g , $\{g(x^n)\}$ is bounded. Taking into account this fact we therefore derive the inequality

$$g(\lambda_n x^n - \lambda x) \leq |\lambda_n - \lambda| g(x^n) + |\lambda| g(x^n - x)$$

which tends to zero as $n \rightarrow \infty$. Hence, the scalar multiplication is continuous.

That is to say that g is a paranorm on the space $Ces_M^{(2)}(\Delta, p)$, as asserted. \square

Theorem 3. *The space $Ces_M^{(2)}(\Delta, p)$ is complete with respect to its paranorm.*

Proof. Let $(x^s) = \{x_{ij}^s\}$ be any Cauchy sequence in the space $Ces_M^{(2)}(\Delta, p)$. Since (x^s) is a Cauchy sequence, we have

$$(1) \quad g(x^s - x^t) \rightarrow 0$$

as $s, t \rightarrow \infty$. Hence, we get

$$|\Delta x_{ij}^s - \Delta x_{ij}^t| \rightarrow 0$$

as $s, t \rightarrow \infty$ for all $i, j \in \mathbb{N}$. Then, we have $\{x_{ij}^s\}$ is a Cauchy sequence in \mathbb{R} for each fixed $i, j \in \mathbb{N}$.

Thus, there exists $x_{ij} \in \mathbb{R}$ such that $x_{ij}^s \rightarrow x_{ij}$ as $s \rightarrow \infty$ and say $x = x_{ij}$. Since M is continuous, by (1) we get

$$g(x^s - x) \rightarrow 0$$

as $t \rightarrow \infty$.

Since $Ces_M^{(2)}(\Delta, p)$ is linear space, we get $x = \{x_{ij}\} \in Ces_M^{(2)}(\Delta, p)$. This completes the proof. \square

Theorem 4. *Let $0 < p_{nm} \leq q_{nm} < \infty$. Then $Ces_M^{(2)}(\Delta, p) \subset Ces_M^{(2)}(\Delta, q)$.*

Proof. Let $x \in Ces_M^{(2)}(\Delta, p)$, then there exists $\rho > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{pnm} < \infty.$$

Hence we have $M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) < 1$ for large values of n, m . Then, we get

$$\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{qnm} \leq \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{pnm} < \infty$$

and so $x \in Ces_M^{(2)}(\Delta, q)$. \square

Theorem 5. Let M_1 and M_2 be Orlicz functions satisfying Δ_2 -condition. Then

- (a) $Ces_{M_1}^{(2)}(\Delta, p) \subset Ces_{M_2 \circ M_1}^{(2)}(\Delta, p)$,
- (b) $Ces_{M_1}^{(2)}(\Delta, p) \cap Ces_{M_2}^{(2)}(\Delta, p) \subset Ces_{M_1+M_2}^{(2)}(\Delta, p)$.

Proof. (a) Let $x \in Ces_{M_1}^{(2)}(\Delta, p)$. Then there exists $\rho > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty.$$

Since M_1 is a continuous function, we can find a real number δ with $0 < \delta < 1$ such that $M_1(t) < \varepsilon$. Let $y_{nm} = M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right)$. Hence we write

$$\sum_{n,m=1}^{\infty} [M_2(y_{nm})]^{p_{nm}} = \sum_{y_{nm} \leq \delta} [M_2(y_{nm})]^{p_{nm}} + \sum_{y_{nm} > \delta} [M_2(y_{nm})]^{p_{nm}}.$$

By the properties of M_2 , we have

$$(2) \quad \sum_{y_{nm} \leq \delta} [M_2(y_{nm})]^{p_{nm}} \leq \max \{1, M_2(1)^H\} \sum_{y_{nm} \leq \delta} [y_{nm}]^{p_{nm}}.$$

Also,

$$M_2(y_{nm}) < M_2 \left(1 + \frac{y_{nm}}{\delta} \right) < \frac{1}{2} M_2(2) + \frac{1}{2} \left(\frac{2y_{nm}}{\delta} \right)$$

for $y_{nm} > \delta$. Since M_2 satisfying Δ_2 -condition and $\frac{y_{nm}}{\delta} > 1$, there exists $T > 0$ such that

$$M_2(y_{nm}) < \frac{1}{2} T \frac{y_{nm}}{\delta} M_2(2) + \frac{1}{2} T \frac{y_{nm}}{\delta} M_2(2) = T \frac{y_{nm}}{\delta} M_2(2).$$

Therefore we have

$$(3) \quad \sum_{y_{nm} > \delta} [M_2(y_{nm})]^{p_{nm}} \leq \max \left\{ 1, \left(T \frac{M_2(2)}{\delta} \right)^H \right\} \sum_{y_{nm} > \delta} [y_{nm}]^{p_{nm}}.$$

Hence by the (2), (3), we get

$$\begin{aligned} \sum_{n,m=1}^{\infty} \left[(M_2 \circ M_1) \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} &= \sum_{n,m=1}^{\infty} [M_2(y_{nm})]^{p_{nm}} \\ &\leq B \cdot \sum_{y_{nm} \leq \delta} [y_{nm}]^{p_{nm}} \\ &\quad + F \cdot \sum_{y_{nm} > \delta} [y_{nm}]^{p_{nm}} \\ &< \infty, \end{aligned}$$

where $B = \max \{1, M_2(1)^H\}$ and $F = \max \left\{ 1, \left(T \frac{M_2(2)}{\delta} \right)^H \right\}$. Hence $Ces_{M_1}^{(2)}(\Delta, p) \subset Ces_{M_2 \circ M_1}^{(2)}(\Delta, p)$.

(b) Let $x \in Ces_{M_1}^{(2)}(\Delta, p) \cap Ces_{M_2}^{(2)}(\Delta, p)$. Then there exists $\rho > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty$$

and

$$\sum_{n,m=1}^{\infty} \left[M_2 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty.$$

Hence we get

$$\begin{aligned} & \sum_{n,m=1}^{\infty} \left((M_1 + M_2) \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ & \leq A \cdot \sum_{n,m=1}^{\infty} \left(M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ & \quad + A \cdot \sum_{n,m=1}^{\infty} \left(M_2 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\Delta x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ & < \infty, \end{aligned}$$

where $A = \max \{1, 2^{H-1}\}$. This completes the proof. \square

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